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Abstract

The η-factor is of principal interest to derive the fracture toughness of elastic plastic materials from a load displacement record. This paper describes how this η-factor can be experimentally determined and theoretically derived for a circumferentially-Cracked Round Bar (CRB).

To derive the η-factor, the limit load function is established using finite element calculations based on the Markov theorem.

Experimental results obtained on precracked specimens corroborate the limit load function and the η-factor.

Key words: circumferentially-cracked round bar, fracture toughness, limit load, η-factor, J-integral, finite element analysis, 18MND5

1 Introduction

Testing methods to measure the fracture toughness in the lower and the upper shelf regions are now well-established on a limited number of specimen geometries and are described in different standards. However, to obtain valid measurements, size criteria should be verified. As a result, the specimen size to be tested is large in the lower and the transition region. For example, the recommendation of the ASTM Standard Test Method for Plane Strain Fracture Toughness of Metallic Materials (E 399-83) is:

(1) $$B,\ a \geq 2.5 \left(\frac{K_{Ic}}{\sigma_{YS}}\right)^2$$

where B is the specimen thickness, a the crack length, σ_{YS} the yield strength and K_{Ic} the plane-strain fracture toughness.

It was thus suggested to apply the J-integral based on an Elastic Plastic Fracture Mechanic (EPFM) theory, already used to measure the resistance to initiation of stable crack growth in metals (J_{Ic}), to measure the onset of cleavage (J_c). The fracture toughness is related to the J-integral through K_{Jc}, defined as:

(2) $$K_{Jc} = \sqrt{\frac{J_c\, E}{1 - \nu^2}}$$

where E is the Young's modulus and ν the Poisson's ratio.

However, cleavage is a stress-controlled fracture mechanism and is very sensitive to loss of constraint. On this basis, a size requirement has recently been integrated in the ASTM Standard Test Method for J-integral Characterization of Fracture Toughness (E1737-96):

(3) $$B,\ b,\ a \geq 200\ \frac{J_c}{\sigma_Y}$$

where b is the uncracked ligament length and σ_Y the effective yield strength calculated as the average of the yield strength σ_{YS} and the ultimate tensile strength σ_{TS}.

The E1737-96 standard is based on the evaluation of the J-integral. However, this J-integral is reduced to a simple analytical relation allowing to derive it from a load displacement record. As the CRB geometry (see Figure 1) is not yet mentioned in existing standard and seems to be promising in the field of the evaluation of the fracture toughness using small specimens [1], the authors have developed in this paper an analytical relation for the J-integral based on the η-factor.

2 η-factor formulation

The J-integral formula introduced by Rice requires the knowledge of the stress and strain field on a path around the crack. The evaluation of this formula needs time consuming finite element calculations. On the other hand, the J-integral is also interpreted as an energy released rate per unit of crack area:

(4) $$J = \frac{K^2(1-\nu^2)}{E} + \frac{1}{2\pi b}\left.\frac{\partial U_{pl.}}{\partial b}\right|_{\delta_{pl.}} \quad \text{with}$$

$$U_{pl.} = \int_0^{\delta_{pl.}} P\, d\delta_{pl.}$$

where K is the stress intensity factor, $\delta_{pl.}$ is the plastic part of the displacement under external load P and $U_{pl.}$ is the plastic energy dissipated during the test.

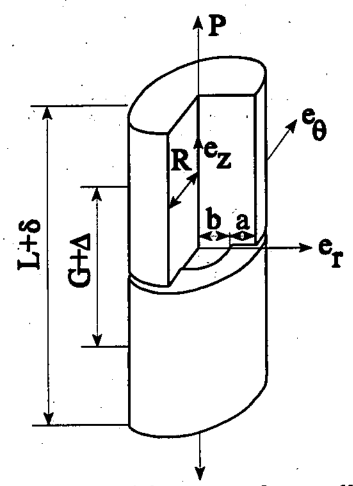

Figure 1: *Geometry of the circumferentially-cracked round bar deformed by a tensile load. δ is the load point displacement and Δ gives the gauge length displacement.*

For practical evaluation of the J-integral from the experimental record of load and displacement, an η-factor is defined [2]:

(5) $$\eta = \frac{b}{2U_{pl.}}\left.\frac{\partial U_{pl.}}{\partial b}\right|_{\delta_{pl.}} = \left.\frac{\partial \ln(U_{pl.})}{2\,\partial \ln(b)}\right|_{\delta_{pl.}}$$

The substitution of Eq. 5 in Eq. 4 gives a practical formula to evaluate the fracture toughness:

(6) $$J = \frac{K^2(1-\nu^2)}{E} + \eta\,\frac{U_{pl.}}{\pi b^2}$$

For a rigid perfectly plastic material, it is easy to establish a relation between the η-factor and the limit load (P_L).

(7) $$\eta = 0.5\,\frac{b}{P_L}\left.\frac{\partial P_L}{\partial b}\right|_{pl.}$$

For convenience a dimensionless limit load (p_L) is defined:

(8) $$p_L = \frac{P_L}{\pi\, b^2\, \sigma_{YS}}$$

Using the limit load presented by Miller [3]:

(9) $$p_L = \begin{cases} 2.85 & \text{for} \quad a/R > 0.65 \\ \dfrac{R}{b} & \text{for} \quad a/R < 0.65 \end{cases}$$

, the η-factor is:

(10) $$\eta = \begin{cases} 1 & \text{for} \quad a/R > 0.65 \\ 0.5 & \text{for} \quad a/R < 0.65 \end{cases}$$

It should be noted that the limit load is exact for a deep crack according to the Tresca criterion and is an approximate linear interpolation for a/R<0.65. This explains the non-physical discontinuity of the η-factor.

To improve the quality of the limit load solution, finite element calculations using the Von Mises criterion are performed with ELSA software [4]. The algorithm is based on the modified Markov variational principle. Thus, the calculation of the limit load becomes a mathematical programming problem. In comparison to step-by-step computations, this direct limit-state analysis offers considerable time saving and high efficiency. Moreover, it overcomes a numerical difficulty linked to plastic incompressibility.

The 8-nodes isoparametric element is adopted in combination with a special degenerated element at the crack tip. This degenerated element has a 1/r strain singularity [5], which satisfies the crack tip singularity of HRR field for a perfectly plastic material. Only few elements (about 30) are needed to obtain a convergent result, and the error is estimated to be within 1%. Present results are also in good agreement with O'Dowd's finite element solution, where the step-by-step calculation with Von Mises' criterion is carried out [6].

The limit load results presented in Figure 2 have been translated into explicit formula imposing the continuity and the derivability at *a/R=0.7* (Eq. 11). This corresponds to the transition between two plastic slipping mechanisms, namely, completely confined plasticity into the ligament for a very deep crack and large scale yielding for a shallow crack.

(11) $$p_L = 3 \qquad \text{for} \quad a/R > 0.7$$

$$p_L = -1.497 + 3.11352\,\frac{R}{b} - 0.6539\,\left(\frac{R}{b}\right)^2 + 0.03738\,\left(\frac{R}{b}\right)^3 \qquad \text{for} \quad a/R < 0.7$$

Substituting Eq. 11 in Eq. 5 gives (see Figure 3):

(12) $$\eta = 1 \qquad \text{for} \quad a/R > 0.7$$

$$\eta = \frac{-1.497 + 1.55676\,\frac{R}{b} - 0.01869\,(\frac{R}{b})^3}{-1.497 + 3.11352\,\frac{R}{b} - 0.6539\,(\frac{R}{b})^2 + 0.03738\,(\frac{R}{b})^3}$$

$$for \quad a/R < 0.7$$

Figure 2 shows a difference between finite element results and Miller's equation (Eq. 9). For deep cracks the 5% difference is due to the fact that the Von Mises criterion allows 0 to 15% higher limit loads [3].

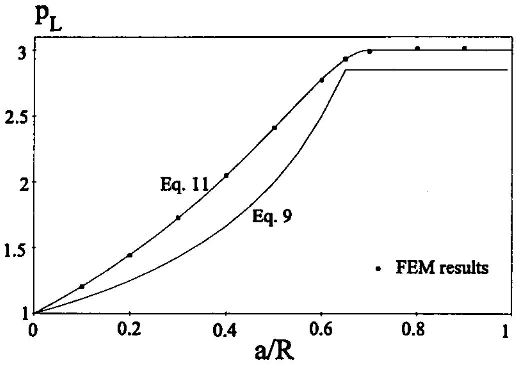

Figure 2: *Dimensionless limit load as a function of the crack length. Finite element results are obtained from the modified Markov theorem (Von Mises criterion). Convergency analysis shows that these results are within 1% accuracy.*

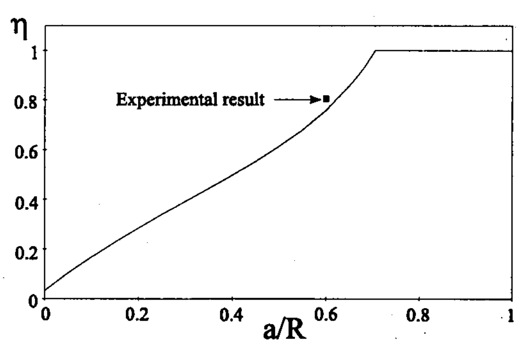

Figure 3: η*-factor derived from finite element results as a function of the crack length (Eq. 12).*

3 Experimental results

This experimental study is performed at ambient temperature with a typical French pressure vessel steel 18MND5 (AFNOR standard). Its chemical composition and tensile properties are given in Table 1 and 2.

C	Si	P	S	Cr	Mn	Ni	Cu	Mo
0.18	0.26	0.007	0.002	0.18	1.55	0.65	0.14	0.5

Table 1: *Chemical composition (wt%).*

Orientation	σ_{YS} (MPa)	σ_{TS} (MPa)	ε_{TS} (%)	δ (%)	RA (%)
L	518	661	12	24	76

Table 2: *Tensile properties at ambient temperature. δ is the total elongation at rupture, ε_{TS} is the elongation at the ultimate tensile strength and RA is the Reduction of Area.*

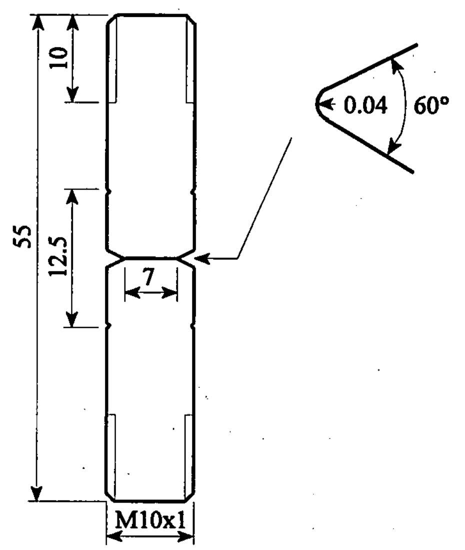

Figure 4: *Specimen size; the threads are machined after precracking the specimen.*

In order to experimentally determine the η-factor, five specimens with the same geometry, see Figure 4, are precracked using the rotating bending fatigue technique [1]. The specimens are precracked up to different crack length to radius ratios. The specimens are loaded in tension with an INSTRON tensile machine at a constant crosshead speed of 0.2mm/min. The load is measured using a 100kN load cell and the elongation of the specimen is determined through two extensometers mounted in opposition with a gauge length of 12.5mm [1]. Table 3 presents the different measured crack lengths and the ratio between the recorded maximal load and the calculated limit load. The accordance between numerical and experimental results is very satisfactory in spite of the experimental scatter and difference between the behaviour of a perfectly plastic material and a real strain-hardening one. Figure 5 shows the load-displacement traces used to derive the plastic energy dissipated as a function of the crack length for a plastic displacement of 0.05 and 0.16 mm (see Figure 6). In accordance with Eq. 5 the slope of the line calculated by the least square method gives an η-factor of 0.79 and is representative for an average crack length to radius ratio of 0.599. This result is in good agreement (see Figure 3) with the theoretical η-factor derived from the limit load function (η=0.755 for a/R=0.6). Moreover, the experimental results show a very small effect of the load level on the η-factor. The drawback of this experimental technique to determine the η-factor is that a very large number of results with crack length to radius ratio from 0 to 1 are needed in order to establish the effect of the crack length to radius ratio on the η-factor. From a technical point of view, the developed technique to precrack round notched bars [1] does not allow yet the generation of very short cracks.

Spec.	a/R	P_{max}/P_L
1	0.767	1.10
2	0.677	1.04
3	0.588	1.00
4	0.520	1.06
5	0.443	1.13
average	0.599	1.07

Table 3: *Test results. P_{max} is the maximum load recorded during the test and P_L is the limit load calculated using Eq. 11.*

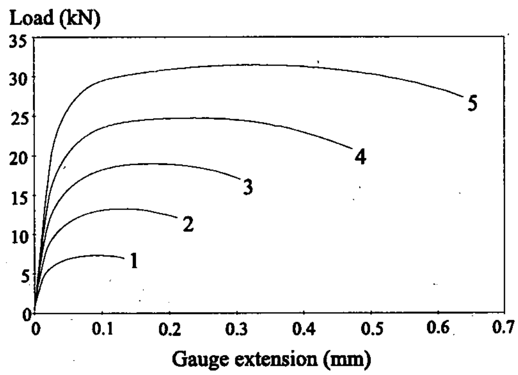

Figure 5: *Load displacement traces.*

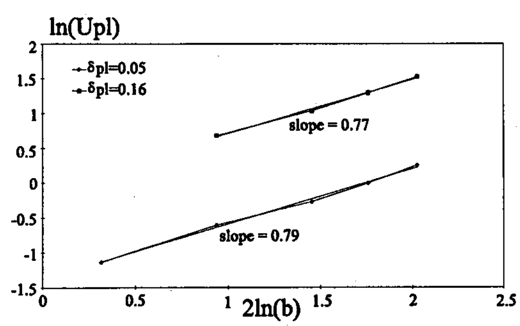

Figure 6: η-*factor is derived experimentally using the least square fit of the experimental data. In comparison,* the η-*factor from Eq. 12 is 0.755 for a/R=0.6.*

4 Conclusion

The application of the Miller limit load function for a CRB of a rigid perfectly plastic material leads to a non physical discontinuity of the η-factor.

Present finite element calculation allows us to derive an explicit formula of the limit load and of the η-factor for a perfectly plastic material.

The limit load and the η-factor function developed in this study are corroborated by experimental results. However, due to the limited range of experiments, the effect of the crack length to radius ratio on the η-factor is not yet established experimentally.

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